Causal Inference Chap 3: A Classification of Assignment Mechanisms

Jinwon Park

July 7, 2022

Seoul National University

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Introduction

Fundamental problem: presence of missing data/ Key component of Causal Inference: Assignmnet Mechanism.

- definition

  - The process that determines which units receive which treatments

  - Which potential outcomes are realized and can be observed?

  - Which potential outcomes are missed?

- Three restrictions
  1. Individualistic assignment
  2. Probabilistic assignment
  3. Unconfounded assignment

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Notation

- N: number of population units. N_c number of controlled units. N_t number of active units
- k: number of covariates or attributes
- X: $N \times k$ matrix of covariates in the population
- $Y_i(0)$, $Y_i(1)$: individual potential outcomes. 0 as under control treatment. 1 as under active treatment.

  * SUTVA: Stable Unit Treatmenet Value Assumption
- W: N-component column vector of treatment assignments with ith element $W_i \in \{0, 1\}$

Notation

$$Y_i(0) = \begin{cases} Y_i^{mis} & \text{if } W_i = 1 \ , \\ Y_i^{obs} & \text{if } W_i = 0, \end{cases} \quad \text{and} \quad Y_i(1) = \begin{cases} Y_i^{mis} & \text{if } W_i = 0 \\ Y_i^{obs} & \text{if } W_i = 1 \end{cases}$$

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Assignment Probabilities

- Given a population of N units, assignment mechanism is a function $Pr(W|X, Y(0), Y(1))$ taking values in $[0, 1]$

$$\sum_{W \in \{0,1\}^N} Pr(W|X, Y(0), Y(1)) = 1$$

- probabilities across the full set of 2^N possible assignment vectors
- Unit assignment probability

$$p_i(X, Y(0), Y(1)) = \sum_{W: W_i = 1} Pr(W|X, Y(0), Y(1))$$

$\rightarrow$ The probability that unit i is assigned to active treatment

Assignment Probabilities

- Propensity score

  $\rightarrow$ average unit assignment probability for units with $X_i = x$

$$e(x) = \frac{1}{N(x)} \sum_{i:X_i=x} p_i(X, Y(0), Y(1))$$

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Restrictions on the assignment mechanism

1. Individualistic Assignment
2. Probabilistic Assignment
3. Unconfounded Assignment

Individualistic Assignment

- Limits the dependence of the treatment assignment for unit i on the outcomes and assignments for other units.

  For some function $q(\cdot) \in [0, 1]$,

  $$p_i(X, Y(0), Y(1)) = q(X_i, Y_i(0), Y_i(1)), \text{ for all } i = 1, \cdots N$$

  and

  $$Pr(W|X, Y(0), Y(1))$$
  $$= c \cdot \prod_{i=1}^{N} q(X_i, Y_i(0), Y_i(1))^{W_i} (1 - q(X_i, Y_i(0), Y_i(1)))^{1-W_i}$$

Probabilistic Assignment

- Every unit to have positive probability of being assigned to treatment level 0 or 1

$$0 < p_i(X, Y(0), Y(1)) < 1, \text{ for each possible } X, Y(0), Y(1)$$

for all $i = 1, \cdots, N$

Unconfounded Assignment

- Restriction on the dependence of the assignment mechanism on potential outcomes

$$Pr(W|X, Y(0), Y(1)) = Pr(W|X, Y^{'}(0), Y^{'}(1))$$

- If individualistic and unconfounded assignment both satisfied,

$$\begin{aligned} Pr(W|X, Y(0), Y(1)) &= Pr(W|X) \\ &= c \cdot \prod_{i=1}^{N} q(X_i)^{W_i} \cdot (1 - q(X_i))^{1-W_i} \end{aligned}$$

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Frame Title

- Taxonomy of assignment mechanism as the organizing principle
  1. Individualistic assignment
  2. Probabilistic assignment
  3. Unconfoundness
- Regular assignment mechanisms are defined by using three restrictions.